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Coherent response in the forward direction to the almost stepwise γ-pulse

E A Popov

Zavoisky Physicotechnical Institute, Sibirskii Trakt 10/7, Kazan, RU-420029, Tatarstan, Russia

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Abstract. The coherent response in the forward direction to the almost stepwise γ-pulse (hereafter one supposes that the pure stepwise γ-pulse is described by a $\theta(t)$ step function) is theoretically analysed. In this way, first of all the analytical form of the γ-pulse has been obtained when 90° remagnetization of the 'magnetic shutter' takes place. The problem is solved by employing the semiclassical Maxwell–Bloch formalism. Then the time evolution of the coherent response to that γ-pulse is calculated when it propagates through a two-level resonant medium. Finally the theoretical results are used to describe the experiment quantitatively.

1. Introduction

The propagation of γ-pulses through a resonant medium has been investigated previously [1, 2] where a γ-pulse was obtained either by the instantaneous mechanical modulation of a natural source or by remagnetization of a perfect $^{57}FeBO_3$ crystal included in the experimental set-up (the so-called 'magnetic shutter' (MS)). Recently the propagation of an almost stepwise γ-pulse in the forward direction has been experimentally researched using a MS [3]. The experimental set-up contained two $^{57}FeBO_3$ crystals: the first acted as the polarization of Mössbauer source radiation and the second was transparent to the incident γ-quanta prior to t_1. At t_1 the resonant interaction of the γ-quanta with the shutter was switched by the 90° remagnetization of this $^{57}FeBO_3$ crystal in the time interval $\Delta t \leqslant 5$ ns, which is much less than the lifetime of the first excited nuclear state of ^{57}Fe ($\tau_N \simeq 141$ ns), and the quanta are effectively absorbed. The γ-pulse obtained in this manner fell on the nuclear target $K_2Mg^{57}FeCN_8 \cdot 10H_2O$ which has a single absorption line. All elements of the set-up are in resonance with the γ transition $(1/2)_g \rightarrow (1/2)_e$ of the shutter. One needs to note that the time in this case is defined by t_1 unlike the general coincidence scheme [4] where the time t_0 is connected to the formation of the nuclear state of 14.4 keV energy in a ^{57}Co source. The experimental form of the coherent response observed at $t \geqslant t_1 = 0$ is the flash ($t_{max} \simeq$ 20–25 ns) of approximate duration 100 ns which has an intensity comparable with that of the incident γ-pulse. This behaviour of the coherent response is different from the results of nuclear forward-scattering experiments with synchrotron radiation [5, 6] and needs a theoretical description, although it is necessary to note that a qualitative consideration connected with the 'nuclear exciton' concept has been proposed in [7].

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2. Theoretical formalism

For the first time let us obtain the form of the γ-pulse which appears after the propagation of incident radiation through a MS during its 90° remagnetization. To solve this problem the semiclassical Maxwell–Bloch formalism (MBF) describing the interaction of a radiation pulse of low intensity with an optically dense medium is used [8]. The MBF gives results like those obtained from the general theory of coherent Mössbauer scattering [9, 10] when the nuclear transition currents are time independent and it is more convenient when nuclear γ-transitions are influenced by time-dependent perturbations (for instance, both sample remagnetization and double γ-NMR resonance). So the corresponding shortened Maxwell–Bloch equation system is [11]

$$\begin{aligned}
&\frac{\partial \boldsymbol{a}}{\partial z} + \frac{1}{c}\frac{\partial \boldsymbol{a}}{\partial t} = \frac{2\pi \mathrm{i}}{kc}\sum_{e,g}\sum_{n} \boldsymbol{j}^{ge}\sigma^{eg}\delta(\boldsymbol{r}-\boldsymbol{r}_n) \\
&\frac{\partial \sigma^{eg}}{\partial t} = \mathrm{i}\left(\Delta - \omega_{eg} + \frac{\mathrm{i}\Gamma}{2}\right)\sigma^{eg} - \frac{\mathrm{i}}{h}\sum_{n}\rho_g^{(0)}\boldsymbol{j}^{eg}\cdot \boldsymbol{a}\delta(\boldsymbol{r}-\boldsymbol{r}_n)
\end{aligned} \tag{1}$$

where $\boldsymbol{a}$ and σ^{eg} are the convolutions of the γ-quanta field and of the matrix elements of density matrix of the nuclear system, respectively; $\boldsymbol{j}^{eg}$ are the matrix elements of the current operator of γ-transitions; Δ is the Doppler shift of incident γ-quanta; ω_{eg} are the γ-transition frequencies when there is only hyperfine splitting of nuclear levels; Γ is the natural width of the Mössbauer line; $\rho_g^{(0)} = 1/2I_g + 1$ is the equilibrium population of the ground nuclear state. Here the general plane-wave approximation is used when $\boldsymbol{a}$ depends on z only. Passing to the Fourier images

$$\begin{aligned}
\boldsymbol{a}(z,t) &= \int_{-\infty}^{\infty} \mathrm{d}\nu\, \boldsymbol{a}(z,\nu)\exp[-\mathrm{i}\nu t] \\
\sigma^{eg}(z,t) &= \int_{-\infty}^{\infty} \mathrm{d}\nu\, \sigma^{eg}(z,\nu)\exp[-\mathrm{i}\nu t]
\end{aligned}$$

one can obtain from (1)

$$\frac{\partial a_p}{\partial z} + \mathrm{i}\frac{\nu}{c}a_p = -\frac{2\pi f\eta n}{hk(2I_g+1)}\sum_{p'}\sum_{e,g}\frac{B^{*}_{eg^{(p)}}B_{eg^{(p')}}a_{p'}}{\Delta + \nu - \omega_{eg} + \mathrm{i}\frac{\Gamma}{2}} \tag{2}$$

where a_p are the transverse components of the electromagnetic field; η is the abundance of the Mössbauer isotope; n is the nuclei number in the unit volume, $B_{eg^{(p)}} = (-\boldsymbol{j}^{eg}\cdot \boldsymbol{e}_p/c)\exp[\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}]$.

It is known [12] that

$$B_{eg^{(p)}} = (2\pi)^{1/2}\sum_{L=1}^{\infty}\sum_{M=-L}^{L}\mathrm{i}^L(2L+1)^{1/2}D^{(L)}_{Mp}(\varphi,\theta)[\boldsymbol{A}_{LM}(m) + \mathrm{i}p\boldsymbol{A}_{LM}(e)]\cdot \boldsymbol{j}^{eg}$$

for the circular polarization basis ($\boldsymbol{e}_p = (\boldsymbol{e}_x + \mathrm{i}p\boldsymbol{e}_y)/2^{1/2}$, $p = \pm 1$). Here $\boldsymbol{A}_{LM}(m)$ is the multipole of a magnetic γ-transition and $\boldsymbol{A}_{LM}(e)$ is the multipole of an electric γ-transition; $D^{(L)}_{Mp}(\varphi,\theta)$ are rotating matrices of the wavevector of the incident γ-quanta in the eigencoordinate system. Let us consider now the magnetodipole γ-transition ($L = 1$). That is right for instance for the γ-transitions $I_g = 1/2 \rightarrow I_e = 3/2$ of the ^{57}Fe isotope. If $T_{1M} = (-1/c)\boldsymbol{j}\cdot\boldsymbol{A}_{1M}(m)$, then $\langle I_e m_e|T_{1M}|I_g m_g\rangle = C(I_g 1 I_e m_g M m_e)\langle I_e\|T_1\|I_g\rangle$ because of the Wigner–Eckhart theorem, and also $\langle I_e\|T_1\|I_g\rangle = (\Gamma/8\pi(1+\alpha))^{1/2}$ [9], where α is the conversion coefficient. Substitution of these expressions into (2) gives

$$\frac{\partial a_p}{\partial z} + \mathrm{i}\frac{\nu}{c} a_p = -\frac{3\mu}{16} \sum_{M=-1}^{1} \sum_{p'=\pm 1} \sum_{e,g} D^{*(1)}_{Mp}(\varphi,\theta) D^{(1)}_{M'p'}(\varphi,\theta) \times \frac{C^2(I_g 1 I_e m_g M m_e) a_{p'}}{\Delta + \nu - \omega_{eg} + \mathrm{i}\frac{\Gamma}{2}} \tag{3}$$

where $\mu = \Gamma f \eta n \sigma_{abs}$ is the nuclear absorption coefficient.

Let a natural radioactive source be used as the incident radiation source:

$$\boldsymbol{a}_{inc}(t) = \sum_{p=\pm 1} \boldsymbol{e}_p^{inc} \exp\left(\frac{-\Gamma_s(t-t_0)}{2}\right)$$

where $\boldsymbol{e}_p^{inc}$ is the polarization vector of incident radiation and Γ_s the linewidth of the source ($\Gamma_s \simeq \Gamma$). Then, selecting the γ-transition $(1/2)_g \to (1/2)_e$ ($\Delta = \omega_{1/21/2}$), we obtain the equation system describing the propagation of incident radiation through a MS in the two-level resonant approximation:

$$\frac{\partial a_p}{\partial z} + \mathrm{i}\frac{\nu}{c} a_p = \frac{-3/\mu}{16} \sum_{p'=\pm 1} D^{*(1)}_{0p}(\varphi,\theta) D^{(1)}_{0p'}(\varphi,\theta) \frac{C^2(I_g 1 I_e 1/201/2) a_{p'}}{\nu + \mathrm{i}\frac{\Gamma}{2}}$$
$$a_p(\nu, z=0) = 1/\nu + \mathrm{i}\Gamma_s/2 \qquad p = \pm 1. \tag{4}$$

When $t < t_1$ the local hyperfine field H_{hf} is directed along OZ for both elements of the MS, and γ-quanta propagate along OY($\theta = \pi/2, \varphi = \pi/2$). Then equation (4) for the circular polarization basis is

$$\partial a_1/\partial z + \mathrm{i}(\nu/c) a_1 = (-\mu/8)(a_1 - a_{-1})/\nu + \mathrm{i}\Gamma/2$$
$$\partial a_{-1}/\partial z + \mathrm{i}(\nu/c) a_{-1} = (-\mu/8)(a_{-1} - a_1)/\nu + \mathrm{i}\Gamma/2. \tag{4a}$$

For a linear polarization basis ($a_\pi = (a_1 + a_{-1})/2^{1/2}$, $a_\sigma = -\mathrm{i}(a_1 - a_{-1})/2^{1/2}$) it follows from (4*a*) that

$$\partial a_\pi/\partial z + \mathrm{i}(\nu/c) a_\pi = 0$$
$$\partial a_\sigma/\partial z + \mathrm{i}(\nu/c) a_\sigma = (-\mu/4) a_\sigma/\nu + \mathrm{i}\Gamma/2. \tag{4b}$$

It is seen from (4*b*) that both elements of the MS are transparent for the π component of the radiation field and they absorb its σ component. The solution of (4*b*) for $a_\pi(z,t)$, $a_\sigma(z,t)$ can be found from (4*b*) by using the inverse Fourier transformation [4, 8]:

$$a_\pi \sim \exp[-\Gamma_s(t-t_0)/2] \qquad a_\sigma \sim J_0((\beta_i(t-t_0))^{1/2}) \exp[-\Gamma(t-t_0)/2]$$

where J_k ($k = 0, \pm 1, \pm 2 \ldots$) are the Bessel functions of the first kind, $\beta_i = \mu L_i$, L_i are the thicknesses of the polarizer ($i = pl$) and shutter ($i = sh$), respectively. If $\mu L_{pl} \gg 1$, then the σ component of the radiation field is supposed to be absolutely absorbed by the polarizer and only π-polarized γ-quanta fall on the shutter. Let 90° remagnetization of the shutter take place at t_1. In this case the hyperfine interaction Hamiltonian $\mathcal{H}_{hf} = -\sum_j \mu_j H_{hf} I_{jz}$ (μ_j is the nuclear dipole moment, and I_{jx}, I_{jy} and I_{jz} are the projections of the nuclear angular moment, where $j = e, g$) is transformed to $\mathcal{H}'_{hf} = -\sum_j \mu_j H_{hf} I_{jx}$ in the time interval 2–5 ns which is much less than the frequency of Larmor precession of nuclear spin (hereafter we shall suppose that it happens instantaneously). It is known that the process is non-adiabatic when the Hamiltonian $\mathcal{H}_{hf}$ has already changed but the wavefunctions Ψ_{k_j} ($k; = m_e, m_g$) remain former. Now Ψ_{k_j} are not the eigenfunctions of the new Hamiltonian $\mathcal{H}'_{hf}$ but they are the superpositions of its eigenfunctions $\Phi^{(j)}_{\ell_j}$: $\Psi_{k_j} = \sum_{\ell_j} d^{(I_j)}_{k_j \ell_j}(\pi/2)\Phi_{\ell_j}$, where $d^{(I_j)}_{k_j \ell_j}(\pi/2)$ are the Wigner d-functions [12].

Now the equation system describing the propagation of the γ-pulse through the shutter after its remagnetization has the form

$$\frac{\partial a_p}{\partial z} + \mathrm{i}\frac{\nu}{c}a_p = \frac{-3\mu}{16}\sum_{p'=\pm 1}\sum_{e,g}\sum_{\substack{e_1,g_1\\ e_2,g_2}} d_{ee_1}^{(I_e)}\left(+\frac{\pi}{2}\right)C(I_g 1 I_e m_{g_1} M m_{e_1})D_{Mp}^{*(1)}(\varphi,\theta)d_{gg_1}^{(I_g)}\left(\frac{\pi}{2}\right)$$
$$\times d_{gg_2}^{(I_g)}\left(+\frac{\pi}{2}\right)C(I_g 1 I_e m_{g_2} M m_{e_2})D_{M'p'}^{(1)}(\varphi,\theta)\frac{d_{ee_2}^{(I_e)}(\pi/2)a_{p'}}{\Delta+\nu-\omega_{eg}+\mathrm{i}\frac{\Gamma}{2}}. \qquad (5)$$

For the two-level resonant approximation ($\Delta = \omega_{1/21/2}$), it follows from (5) that

$$\partial a_\pi/\partial z + \mathrm{i}(\nu/c)a_\pi = (-\mu/4)a_\pi/\nu + \mathrm{i}\Gamma/2. \qquad (5a)$$

So, by comparing (4*b*) and (5*a*), it is seen that the shutter intensively absorbs the π component of γ-radiation after t_1 if $\mu L_{sh} \gg 1$. When $t_1 > t_0$ the continuous solution of (5*a*) has the form

$$a_{\pi_1}(L_{sh}, t, t_0) = C J_0((\beta_{sh}(t-t_1))^{1/2})\exp[-\Gamma(t-t_1)/2]\,\theta(t-t_1)\theta(t_1-t_0).$$

The constant C is defined by equating the solutions of (4*b*) and (5*a*) at $t = t_1$ ($C = \exp[-\Gamma_s(t-t_0)/2]$). If $t_0 > t_1$ a γ-quantum does not 'feel' the shutter remagnetization and

$$a_{\pi_2}(L_{sh}, t, t_0) = J_0((\beta_{sh}(t-t_0))^{1/2})\exp[-\Gamma(t-t_0)/2]\theta(t_0-t_1).$$

So the general solution for the γ-pulse passing through the shutter is

$$a_\pi(L_{sh}, t, t_0) = \exp[-\Gamma_s(t-t_0)/2]\theta(t_1-t)\theta(t_1-t_0) + a_{\pi_1}(L_{sh}, t, t_0) + a_{\pi_2}(L_{sh}, t, t_0). \qquad (6)$$

Equation (6) corresponds to the γ-pulse [3]. It can be found by averaging the intensity $I_\pi(L_{sh}, t, t_0) \sim |a_\pi(L_{sh}, t, t_0)|^2$ over t_0 which is the random value for MS ($\langle\ldots\rangle_{t_0} = \Gamma_s\int_{-\infty}^{t}\mathrm{d}t_0\,(\ldots)$):

$$\langle I_\pi(L_{sh}, t, t_0)\rangle_{t_0} \sim N_\pi\Bigg[\theta(t_1-t) + \theta(t-t_1)J_0^2((\beta_{sh}(t-t_1))^{1/2})\exp\left(\frac{-\Gamma(t-t_1)}{2}\right)$$
$$+\Gamma_s\int_{t_1}^{t}\mathrm{d}t_0\,J_0^2((\beta_{sh}(t-t_0))^{1/2})\exp\left(\frac{-\Gamma(t-t_0)}{2}\right)\Bigg]. \qquad (6a)$$

Here N_π is the number of resonant γ-quanta passing through the polarizer per second.

Now let the γ-pulse (6) fall on a nuclear target. To describe the propagation of that pulse it is convenient to use the response function formalism [13] in which the radiation field after the target is defined as

$$a_{\pi t}(L_t, t, t_0) = \int_{-\infty}^{\infty}\mathrm{d}t'\,G(L_t, t-t')a_\pi(L_{sh}, t', t_0) \qquad (7)$$

where

$$G(L_t, t-t') = \delta(t-t') - (\beta_t/4\Gamma^2(t-t'))^{1/2}J_1((\beta_t(t-t'))^{1/2})\exp[-\Gamma(t-t')/2]$$

is the resonant response function ($\beta_t = \mu_t L_t$; μ_t and L_t are the nuclear absorption coefficient and thickness of the target, respectively).

Substitution of (6) in (7) gives the form of the coherent response after the target.

(1) At $t < t_1$,

$$a_{\pi t}(L_t, t, t_0) = F_0(L_t, t, t_0)$$

$$F_0(L_t, t, t_0) = \int_{t_0}^{t}\mathrm{d}t'\,G(L_t, t-t')\exp\left(-\frac{\Gamma_s(t'-t_0)}{2}\right).$$

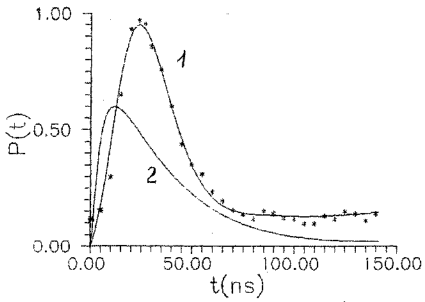

Figure 1. The time dependences $P(t)$ at $t > 0$: curve 1, for the γ-pulse (6a); curve 2, for the stepwise γ-pulse; ——, intensity of the incident γ-pulse at $t < 0$; $*$, experiment [3].

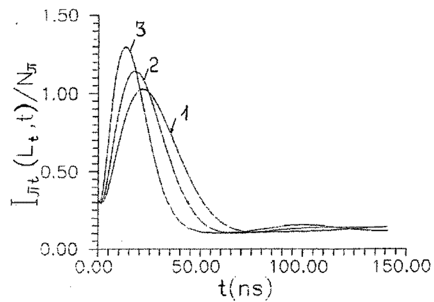

Figure 2. The time dependences $I_{\pi t}(L_t, t)/N_\pi$ at $t > 0$, $\beta_t \simeq 11$: curve 1, $\beta_{sh} \simeq 64$; curve 2, $\beta_{sh} \simeq 84$; curve 3, $\beta \simeq 124$.

(2) At $t \geqslant t_1$,

$$a_{\pi t}(L_t, t, t_0) = \theta(t_1 - t_0)\{F_1(L_t, t, t_0) + F_2(L_t, t, t_0)\} + \theta(t_0 - t_1)F_3(L_t, t, t_0)$$

$$F_1(L_t, t, t_0) = \int_{t_0}^{t_1} \mathrm{d}t'\, G(L_t, t - t') \exp\left(-\frac{\Gamma_s(t' - t_0)}{2}\right)$$

$$F_2(L_t, t, t_0) = \exp\left(-\frac{\Gamma_s(t_1 - t_0)}{2}\right) \int_{t_1}^{t} \mathrm{d}t'\, G(L_t, t - t') J_0((\beta(t' - t_1))^{1/2}) \times \exp\left(-\frac{\Gamma(t' - t_1)}{2}\right)$$

$$F_3(L_t, t, t_0) = \int_{t_0}^{t} \mathrm{d}t'\, G(L_t, t - t') J_0((\beta(t' - t_0))^{1/2}) \exp\left(-\frac{\Gamma(t' - t_0)}{2}\right).$$

The corresponding expressions for intensity $I_{\pi t}(L_t, t)$ can be obtained by averaging $|a_{\pi t}(L_t, t, t_0)|^2$ over time t_0.

(1) At $t < t_1$,

$$I_{\pi t}(L_t, t) = N_\pi \Gamma_s \int_{-\infty}^{t} \mathrm{d}t_0 \qquad F_0^2(L_t, t, t_0). \tag{7a}$$

(2) At $t \geqslant t_1$,

$$I_{\pi t}(L_t, t) = N_\pi \Gamma_s \left(\int_{-\infty}^{t_1} \mathrm{d}t_0\, [F_1(L_t, t, t_0) + F_2(L_t, t, t_0)]^2 + \int_{t_0}^{t_1} \mathrm{d}t'\, F_3^2(L_t, t, t_0) \right). \tag{7b}$$

3. Numerical calculation and analysis

It is known that the intensity of resonant γ-quanta detected in the experiment is the convolution of the coherent response of the target with the detector time function f_d:

$$\bar{I_{\pi t}}(L_t, t) = \int_{-\infty}^{\infty} \mathrm{d}t'\, I_{\pi t}(L_t, t - t') f_d(t'). \tag{8}$$

To fit (8) to the experiment [3] ($\beta \simeq 84\Gamma$; $\beta_t \simeq 11\Gamma$; $\Gamma_s \simeq 2.06\Gamma$; time resolution of the detector, about 5.1 ns) the form of f_d was selected arbitrarily under the condition FWHM = 5.1 ns. The good correlation of the calculated dependence with the experimental data was when

$$f_d = \begin{cases} 0 & t' < 0 \\ a\{(t'/t_2)^2\} & 0 \leqslant t' \leqslant t_2 \\ 1/((2(t' - t_2)/\Delta t)^2 + 1) & t' > t_2 \end{cases}$$

where $t_2 \simeq 0.4$ ns, $\Delta t \simeq 9.6$ ns and $a = 1/(t_2/3 + \pi \Delta t/4)$ is the normalization coefficient.

The time dependences of $P(t) = \bar{I_{\pi t}}(t)/N_\pi$ are drawn in figure 1. One can see that the detected signal for $t > 0$ is the flash with a maximum at $t \simeq 24$ ns and duration about 100 ns. Its maximal value is comparable (about 0.97) with the incident pulse. This behaviour of the theoretical curve corresponds to the result [3] within the statistical error of the experiment. The flash is due to the concrete form of the incident pulse (6*a*) and it is mainly defined by the competition of $F_1(L_t, t, t_0)$ and $F_2(L_t, t, t_0)$ (figure 1, curve 1). If the incident pulse is the pure stepwise function $N_\sigma \Gamma_s \theta(t_1 - t)$, then the flash is absent and the form of the re-emitted signal is defined by $F_1(L_t, t, t_0)$ only (figure 1, curve 2). It is necessary to note that this time evolution is unlike that for the 180° remagnetization case [14] where the form of the resonant response is strongly influenced by the beats due to the interference between the incident radiation and re-emitted radiation of the target.

4. Conclusion

We considered the propagation in a two-level resonant medium of the γ-pulse obtained when 90° remagnetization of the MS takes place. The time evolution of the γ-radiation field passing through the nuclear target is strongly connected with the coherent superposition of the responses to the single components of the incident γ-pulse. It follows from numerical analysis of (7*b*) that the main contribution to the target signal $I_{\pi t}(L_t, t)$ at $t > 0$ is defined by

$$N_\pi \Gamma_s \left(\int_0^\infty \mathrm{d}t_0\, [J_0((\beta_{sh} t)^{1/2}) - A]^2 \right) \exp[-\Gamma t]$$

where

$$A = \left(\frac{\beta_t}{4\Gamma^2} \right)^{1/2} \int_0^{t_0} \mathrm{d}t' \, \frac{J_1((\beta_t (t + t'))^{1/2})}{(t + t')^{1/2}}.$$

As a result, sharp amplification of the coherent response takes place the maximum value of which can exceed the intensity of the incident radiation (figure 2). This could be used for the searching process of γ-laser pumping for short-lived Mössbauer isotopes.

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